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MATDIP401

Fourth Semester B.E. Degree Examination, July/August 2022
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Prove that the sum of the squares of the direction cosines in equal to unity. **(06 Marks)**

b. If $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of a line. Prove that
(i) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
(ii) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$ **(07 Marks)**

c. Find the image of the point (2, –1, 3) in the plane $2x + 4y + z - 24 = 0$. **(07 Marks)**

2 a. Find the equation of the plane in the intercept form. **(06 Marks)**

b. Find the equation of the plane which passes through (3, –3, 1) and is perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. **(07 Marks)**

c. Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$ are coplanar. Find their common point. **(07 Marks)**

3 a. Find sine of the angle between the vectors $2\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} - 2\hat{j} + 2\hat{k}$. **(06 Marks)**

b. Find the constant 'a' such that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar. **(07 Marks)**

c. Prove that $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$. **(07 Marks)**

4 a. A particle moves along the curve $x = 1 - t^3$, $y = 1 + t^2$, $z = 2t - 5$ where t is the time. Find the velocity and acceleration at t = 1. **(06 Marks)**

b. Find the unit normal vector to the surface $xy + x + zx = 3$ at (1, 1, 1). **(07 Marks)**

c. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at the point (2, -1, 2). **(07 Marks)**

5 a. Find the directional derivative of $\phi = x^2yz + xz^2$ at the point $(-1, 2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(06 Marks)**

b. Show that the vectors $\vec{F} = (2xy + z^2)i + (x^2 + 2xy)j + (y^2 + 2zx)k$ is irrotational. **(07 Marks)**

c. Given that $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$, show that $\vec{F}.\text{curl}\,\vec{F} = 0$. **(07 Marks)**

6 a. Using the definition show that $L[t^n] = \frac{n!}{s^{n+1}}$. **(05 Marks)**

b. Find $L[t\cos at]$. **(05 Marks)**

c. Find $L\left[\frac{\cos at - \cos bt}{t}\right]$. **(05 Marks)**

d. Find $L[\cos(at + b)]$. **(05 Marks)**

7 a. Find $L^{-1}\left[\frac{s^2-3s+4}{s^3}\right]$. **(05 Marks)**

b. Find $L^{-1}\left[\frac{s+2}{s^2-4s+13}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s^2+s-2}{s(s+3)(s-2)}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\frac{(s+a)}{(s+b)}\right]$. **(05 Marks)**

8 a. Using Laplace Transform method solve $y''+2y'-3y=\sin t$ subject to the condition, $y(0)=y'(0)=0$. **(10 Marks)**

b. By applying Laplace transform, solve the differential equation $y''+4y'+3y=0$ subject to the condition $y(0)=0$ and $y'(0)=1$. **(10 Marks)**

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